

UK Junior Mathematical Olympiad 2007

Organised by The United Kingdom Mathematics Trust

Tuesday 12th June 2007

RULES AND GUIDELINES : READ THESE INSTRUCTIONS CAREFULLY BEFORE STARTING

1. Time allowed: 2 hours.
2. **The use of calculators and measuring instruments is forbidden.**
3. All candidates must be in *School Year 8 or below* (England and Wales), *S2 or below* (Scotland), *School Year 9 or below* (Northern Ireland).
4. For questions in Section A *only the answer is required*. Enter each answer neatly in the relevant box on the Front Sheet. Do not hand in rough work.

For questions in Section B you must give *full written solutions*, including clear mathematical explanations as to why your method is correct.

Solutions must be written neatly on A4 paper. Sheets must be STAPLED together in the top left corner with the Front Sheet on top.

Do not hand in rough work.

5. Questions A1-A10 are relatively short questions. Try to complete Section A within the first 45 minutes so as to allow well over an hour for Section B.
6. Questions B1-B6 are longer questions requiring *full written solutions*. This means that each answer must be accompanied by clear explanations and proofs. Work in rough first, then set out your final solution with clear explanations of each step.
7. These problems are meant to be challenging! Do not hurry. Try the earlier questions in each section first (they tend to be easier). Try to finish whole questions even if you can't do many. A good candidate will have done most of Section A and given solutions to at least two questions in Section B.
8. Answers must be FULLY SIMPLIFIED, and EXACT using symbols like π , fractions, or square roots if appropriate, but NOT decimal approximations.

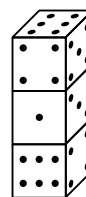
DO NOT OPEN THE PAPER UNTIL INSTRUCTED BY THE INVIGILATOR TO DO SO!

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Section A

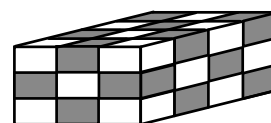
- A1** What is the value of $1^5 - 2^4 + 3^3 - 4^2 + 5^1$?
- A2** What is the value of k if “ $7k$ minutes past nine” is the same time as “ $8k$ minutes to ten”?
- A3** Charlie boils seven eggs for his breakfast. He puts the eggs into the pan one at a time, but waits one minute after putting one egg in before putting the next egg in. If he boils each egg for three minutes, how long does the whole operation take from the moment he puts the first egg in to the moment he takes the seventh egg out?
- A4** The hobbits Frodo, Sam, Pippin and Merry have breakfast at different times. Each one takes a quarter of the porridge in the pan, thinking that the other three have not yet eaten. What fraction of the porridge is left after all four hobbits have had their breakfast?

- A5** The diagram shows a tower consisting of three identical dice.
On these dice, each pair of opposite faces has a total of seven dots.
How many dots are there on the face on which the tower stands?



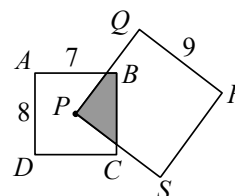
- A6** The sizes in degrees of the interior angles of a pentagon are consecutive whole numbers. What is the size of the largest of these angles?

- A7** A large cuboid is made from cuboids of equal size, coloured alternately black and white, as shown.
What fraction of the surface area of the large cuboid is black?

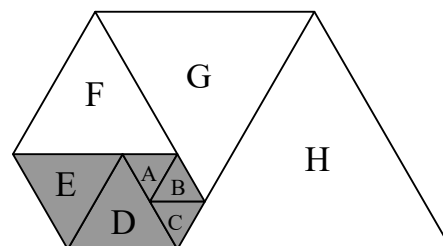


- A8** Pegs numbered 1 to 50 are placed in order in a line with number 1 on the left. They are then knocked over one at a time following these rules:
- Of the pegs which are still standing, knock down alternate ones, starting with the first peg on the left.
 - Each time you reach the end of the row, repeat the previous rule.
- What is the number of the last peg to be knocked over?

- A9** The diagram shows squares $ABCD$ and $PQRS$ of side length 8 units and 9 units respectively. Point P is the centre of square $ABCD$; PQ intersects AB at a point 7 units from A .
What is the perimeter of the shaded region?



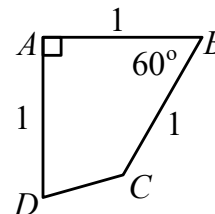
- A10** The diagram shows a spiral of equilateral triangles. After the first five triangles A, B, C, D, E (shown shaded), the next triangle is always placed alongside two others: the one placed immediately before and one placed earlier. The smallest triangles have sides of length 1 unit.
What is the length of the sides of the fifteenth triangle?



Section B

B1 Find four integers whose sum is 400 and such that the first integer is equal to twice the second integer, three times the third integer and four times the fourth integer.

B2 The diagram shows a quadrilateral $ABCD$ in which AB , BC and AD are all of length 1 unit, $\angle BAD$ is a right angle and $\angle ABC$ is 60° .

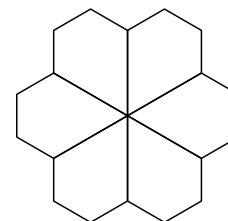


Prove that $\angle BDC = 2 \times \angle DBC$.

- B3** (a) Yesterday evening, my journey home took 25% longer than usual.
By what percentage was my average speed reduced compared to normal?
- (b) By what percentage would I need to increase my usual average speed in order for the journey to take 20% less time than usual?

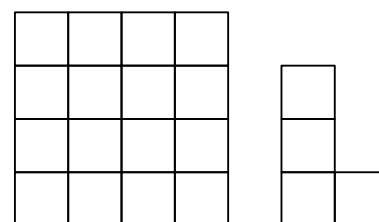
B4 Find a rule which predicts exactly when five consecutive integers have sum divisible by 15.

B5 A window is constructed of six identical panes of glass. Each pane is a pentagon with two adjacent sides of length two units. The other three sides of each pentagon, which are on the perimeter of the window, form half of the boundary of a regular hexagon



Calculate the exact area of glass in the window.

B6 We want to colour red some of the cells in the 4×4 grid shown so that wherever the L-shaped piece is placed on the grid it covers at least one red cell. The L-shaped piece may only cover complete cells, may be rotated, but may not be turned over and may not extend beyond the grid.



- (a) Show that it is possible to achieve this by colouring exactly four cells red.
- (b) Show that it is impossible to achieve this by colouring fewer than four cells red.